Graphical relationship between \( f', f'', \) and \( f''' \)

85 33. The graph of the **derivative** of \( f \) is shown in the figure above. Which of the following could be the graph of \( f' \)?

(A) ![Graph A]
(B) ![Graph B]
(C) ![Graph C]
(D) ![Graph D]
(E) ![Graph E]

69 16. If \( y \) is a function \( x \) such that \( y' > 0 \) for all \( x \) and \( y'' < 0 \) for all \( x \), which of the following could be part of the graph of \( y = f(x) \)?

(A) ![Graph A (69)]
(B) ![Graph B (69)]
(C) ![Graph C (69)]
(D) ![Graph D (69)]
(E) ![Graph E (69)]
17. The graph of a twice-differentiable function $f$ is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

20. The graph of $y = f(x)$ on the closed interval $[2, 7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One  (B) Two  (C) Three  (D) Four  (E) Five

81. Let $f$ be the function given by $f(x) = |x|$. Which of the following statements about $f$ are true?

I. $f$ is continuous at $x = 0$.

II. $f$ is differentiable at $x = 0$.

III. $f$ has an absolute minimum at $x = 0$.

(A) I only  (B) II only  (C) III only  (D) I and III only  (E) II and III only
8. The graph of \( y = f(x) \) is shown in the figure above. On which of the following intervals are \( \frac{dy}{dx} > 0 \) and \( \frac{d^2y}{dx^2} < 0 \)?

- I. \( a < x < b \)
- II. \( b < x < c \)
- III. \( c < x < d \)

(A) I only (B) II only (C) III only (D) I and II (E) II and III

9. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?

I. \[ \begin{align*}
\text{Graph of } y = f(x) & \quad \frac{dy}{dx} > 0 \\
\text{Graph of } y = f'(x) & \quad \frac{d^2y}{dx^2} < 0
\end{align*} \]

II. \[ \begin{align*}
\text{Graph of } y = f(x) & \quad \frac{dy}{dx} < 0 \\
\text{Graph of } y = f'(x) & \quad \frac{d^2y}{dx^2} > 0
\end{align*} \]

III. \[ \begin{align*}
\text{Graph of } y = f(x) & \quad \frac{dy}{dx} > 0 \\
\text{Graph of } y = f'(x) & \quad \frac{d^2y}{dx^2} > 0
\end{align*} \]

(A) I only (B) II only (C) III only (D) I and III (E) II and III
43. Let \( f \) be a function that is continuous on the closed interval \([-2, 3]\) such that \( f'(0) \) does not exist, \( f''(2) = 0 \), and \( f''(x) < 0 \) for all \( x \) except \( x = 0 \). Which of the following could be the graph of \( f' \)?

23. The graph of \( f \) is shown in the figure above. Which of the following could be the graph of the derivative of \( f' \)?
13. The graph of the function $f$ shown in the figure above has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of $x$, $-2 < x < 4$, is $f$ not differentiable?

(A) 0 only  (B) 0 and 2 only  (C) 1 and 3 only  (D) 0, 1, and 3 only  (E) 0, 1, 2, and 3

16. If $y$ is a function of $x$, such that $y' > 0$ for all $x$ and $y'' < 0$ for all $x$, which of the following could be part of the graph of $y = f(x)$?

(A) Y  (B) Y  (C) Y  (D) Y  (E) Y

19. Let $f$ be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \leq 0, \\ x & \text{for } x > 0. \end{cases}$ Which of the following statements about $f$ is true?

(A) $f$ is an odd function.
(B) $f$ is discontinuous at $x = 0$.
(C) $f$ has a relative maximum.
(D) $f'(0) = 0$
(E) $f'(x) > 0$ for $x \neq 0$
11. The graph of the derivative of \( f \) is shown in the figure above. Which of the following could be the graph of \( f \)?

(A) \[ \begin{array}{c}
 y \\
 -2 \quad 0 \quad 2
\end{array} \]

(B) \[ \begin{array}{c}
 y \\
 -2 \quad 0 \quad 2
\end{array} \]

(C) \[ \begin{array}{c}
 y \\
 -2 \quad 0 \quad 2
\end{array} \]

(D) \[ \begin{array}{c}
 y \\
 -2 \quad 0 \quad 2
\end{array} \]

(E) \[ \begin{array}{c}
 y \\
 -2 \quad 0 \quad 2
\end{array} \]

76. The graph of a function \( f \) is shown above. Which of the following statements about \( f \) is false?

(A) \( f \) is continuous at \( x = a \).

(B) \( f \) has a relative maximum at \( x = a \).

(C) \( x = a \) is in the domain of \( f \).

(D) \( \lim_{x \to a^+} f(x) \) is equal to \( \lim_{x \to a} f(x) \).

(E) \( \lim_{x \to a} f(x) \) exists.
The graphs of the derivatives of the functions \( f, g, \) and \( h \) are shown above. Which of the functions \( f, g, \) or \( h \) have a relative maximum on the open interval \( a < x < b \)?

(A) \( f \) only  
(B) \( g \) only  
(C) \( h \) only  
(D) \( f \) and \( g \) only  
(E) \( f, g, \) and \( h \)

The graph of \( y = h(x) \) is shown above. Which of the following could be the graph of \( y = h'(x) \)?

(A)  
(B)  
(C)  
(D)  
(E)